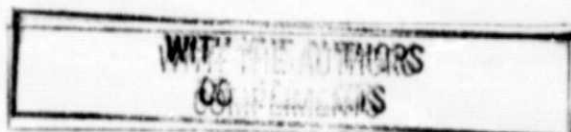


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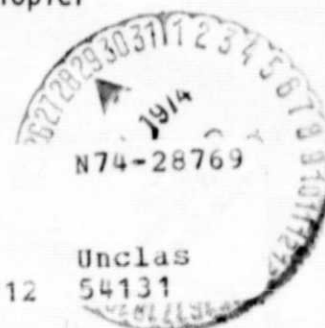
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CONSTANT TEMPERATURE HOT WIRE  
ANEMOMETRY DATA REDUCTION PROCEDURE

G. H. Klopfer

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January 1974

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### ABSTRACT

The theory and data reduction procedure for constant temperature hot wire anemometry are presented. The procedure is valid for all Mach and Prandtl numbers, but limited to Reynolds numbers based on wire diameter between 0.1 and 300. The fluids are limited to gases which approximate ideal gas behavior. Losses due to radiation, free convection and conduction are included.

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## NOMENCLATURE

Most of the nomenclature is listed; those excepted are the numerous factors in the correlation equations which are obvious in the context.

a	non-dimensional temperature
A	$a(\xi) - a^*$
$C_N$	Nusselt number end loss correction factor
$C_R$	recovery ratio end loss correction factor
d	wire diameter (in.)
g	gravitational acceleration, 32.17 ft/sec <sup>2</sup>
Gr	Grasshof number
h	convective heat transfer (BTU/hr-ft <sup>2</sup> -°R)
I	current (amps)
k	thermal conductivity (BTU/hr-ft-°R)
K	unit length resistance (ohms/in.)
Kn	Knudsen number
$\ell$	effective wire length (in.)
M	Mach number
Nu	Nusselt number
p	pressure (lbf/in <sup>2</sup> )
Pr	Prandtl number
$\dot{q}$	heat transfer rate (BTU/hr)
$q_m$	measured hot wire power dissipation (watts/sec)
r	wire resistance (ohms)
R	gas constant (ft-lbf/lbm °R)
S	end loss correction parameter

$T$	temperature ( $^{\circ}\text{R}$ )
$U$	velocity (ft/sec)
$x$	length coordinate (in.)
$z$	non-dimensional length coordinate
$\alpha_1$	temperature coefficient of resistance, ( $^{\circ}\text{R}^{-1}$ )
$\alpha_2$	second temperature coefficient of resistance ( $^{\circ}\text{R}^{-2}$ )
$\beta$	isothermal expansivity coefficient ( $^{\circ}\text{R}^{-1}$ )
$\gamma$	specific heat ratio
$\epsilon$	emissivity
$\zeta$	temperature difference, $T(x) - T_e$
$\eta$	temperature recovery ratio, $T_e/T_s$
$\bar{\eta}$	normalized recovery ratio
$\eta_c$	recovery ratio at continuum limit
$\eta_f$	recovery ratio at free molecule limit
$\theta$	flow direction angle
$\mu$	viscosity (lbm/ft-hr)
$\xi$	non-dimensional length coordinate
$\rho$	density (lbm/ft <sup>3</sup> )
$\sigma$	Stefan-Boltzmann constant
$\omega$	recovery ratio end loss correction parameter

### Subscripts

( ) <sub>a</sub>	air
( ) <sub>c</sub>	conduction
( ) <sub>d</sub>	duct or tunnel walls
( ) <sub>e</sub>	equilibrium
( ) <sub>f</sub>	fluid
( ) <sub>m</sub>	measured

- ( )<sub>r</sub>      radiation
- ( )<sub>s</sub>      wire support
- ( )<sub>t</sub>      total or stagnation
- ( )<sub>wi</sub>     wire
- ( )<sub>0,1,2</sub> reference values except on  $\alpha$
- ( ) <sub>$\infty$</sub>     local free stream if on M, Re, or  $\theta$ ; infinite wire if on Nu
- (  $\overline{\quad}$  )    average



## 1.0 INTRODUCTION

The theory and data reduction procedure for hot wire anemometry are presented in this paper. The basis of hot wire anemometry is that the heat transfer rate is a function of the flow properties of the flow to be measured. By heating the wire above the equilibrium temperature and measuring the current fluctuations to maintain the wire at a constant temperature, the heat loss and hence the flow speed is determined via a series of Mach, Reynolds, and Nusselt (heat transfer coefficient) correlation curves.

The hot wire loses heat by all three heat transfer modes: convection, conduction, and radiation. The effect of each one is considered, since the correlation curves are valid only for the forced convective heat loss. Due to the finite length of the wire, the conduction end losses are considerable, which affects both the heat transfer (convective) and the cold wire equilibrium temperature, which again affects the heat transfer. An analysis of all these "end" losses, free convective and radiative losses is given.

A complete data reduction procedure is given. The computer program and its data input requirements are given. The appendix also presents the non-linear effects of temperature on the wire resistance and a limited uncertainty analysis of the Mach number due to uncertainties in the wire diameter.

The technique for obtaining the data is not presented here. The program listed is quite general, being valid for all types of gases at all Mach numbers for Reynolds numbers based on wire diameter between 0.1 and 300.

## 2.0 HOT WIRE ANEMOMETRY THEORY

The hot wire probe is a device for measuring the local mass flux of a flowing fluid. The probe consists of a very fine wire electrically heated and immersed in a fluid. By measuring the heat transfer rate off the wire, the mass flow can be deduced. The heat transfer rate is easily obtained by measuring the electrical resistance and the voltage drop of the hot wire.

Two principles are the basis of hot wire anemometry: 1) the electrical resistivity of the wire is temperature dependent, preferably linearly, and 2) the heat transfer rate is a function of the flow properties of the flow. The first principle is necessary so that the temperature of the wire needed for the heat transfer determination can be accurately measured. The need for the second principle is obvious.

The hot wire has two modes of operation: constant current and constant temperature. In the constant current mode, the electrical current through the wire is held constant by an electronic circuit. A variation in the mass flow changes the heat transfer, temperature and, thus, resistance of the wire. The instantaneous resistance is measured to obtain the heat transfer rate and the mass flow. In the second mode the current is modulated with the varying flow to maintain it at constant temperature and resistance. The instantaneous current through the wire is measured to obtain the heat loss. The constant temperature mode has several advantages over the constant current mode, among which are simpler and more straightforward data acquisition and reduction procedures. These and other reasons have made the constant temperature method more useful and will be used in this report.

The theoretical analysis of heat transfer from circular cylinders

will not be given here. As yet no complete analysis with all the complications exists. Partial theories have been developed for the free molecular, high Reynolds number, and low Reynolds number flow regimes. For the transition zones empirical data only are available. Extensive empirical data have been published on the heat transfer rates in terms of Nusselts number of infinitely long circular cylinders as a function of the flow properties, namely Mach and Reynolds numbers. These data are reduced into a set of "universal" correlation curves, Fig. 3. The universal correlation curves allow the direct determination of the Mach and Reynolds (based on wire diameter) from the measured heat transfer corrected for the effects of finite wire length.

The finite wire differs from the infinitely long wire in that heat is conducted to the supports of the finite wire in addition to the convected and radiated heat loss. This conduction end loss causes a nonuniform temperature distribution. The infinite wire loses heat only by convection and radiation and therefore has a uniform temperature distribution. The end loss of the finite wire also affects the adiabatic recovery temperature of the wire which also affects the heat transfer rate of the finite wire as compared to the infinite wire.

The end loss effects of the finite wire must, therefore, be carefully taken into account before the correlation curves of Fig. 3 for infinitely long cylinders are applicable. Since the end effects are so important (the end conduction heat loss can be as great as 50% of the total heat loss for wire aspect ratios of 100), the theoretical analyses of the end losses are given in detail below.

## 2.1 Derivation

The system to be analyzed is shown in Fig. 1a and b. Figure 1a

shows the complete wire probe and lb, the wire and coordinate system. The steady state case will be considered only.

The wire segment of length  $\Delta x$  loses heat by four mechanisms, which are forced convection

$$\dot{q}_h = h \pi d \Delta x (T(k) - T_e)$$

where  $h$  = coefficient of convective heat transfer

$T(x)$  = temperature of wire

$T_e$  = equilibrium (adiabatic wall) temperature of wire,

free convection

$$\dot{q}_{f.c.} = h_{f.c.} \pi d \Delta x (T(k) - T_e)$$

where  $h_{f.c.}$  = average free convective heat transfer coefficient

$$\approx 1.16 \frac{k_f}{d} (Gr Pr)^{1/10} \quad (\text{Ref. 21})$$

$$\text{for } 10^{-5} < Gr < 10^3$$

$$\text{and } Gr = \text{Grashoff's number} = \frac{\rho^2 g \beta (T(k) - T_e) d^3}{\mu_f^2},$$

radiation

$$\dot{q}_r = \sigma \epsilon \pi d \Delta x (T(k)^4 - T_d^4)$$

where  $T_d$  = duct temperature,

and

conduction

$$\dot{q}_h = - \pi \frac{d^2}{4} k_{wi} \frac{d^2 T(x)}{dx^2}$$

The heat losses are balanced by the ohmic heating of the wire

$$\dot{q}_{\text{generated}} = r(x) I^2 \Delta x$$

where  $I$  = electrical current through the wire

and

$$\begin{aligned} r(x) &= \text{resistance per unit length of the wire at} \\ &\quad \text{temperature } T(x) \\ &= r_0 [1 + \alpha_1 (T(x) - T_0) + \alpha_2 (T(x) - T_0)^2] \end{aligned}$$

$T_0$  = reference temperature

$r_0$  = reference resistance at  $T_0$

$\alpha_1$  = first temperature coefficient of resistivity

$\alpha_2$  = second temperature coefficient. The influence of  $\alpha_2$  is negligible for small overheat ratios  $[(r(x) - r_e)/r_e]$ , as is shown in Appendix B.

Defining  $\zeta(x) = T(x) - T$  and assuming that  $T_d \sim T_e$  and  $|[T(x) - T_d]/T_d| < 1$  or

$$T(x)^4 - T_d^4 \approx 4 T(x)^3 [T(x) - T_e]$$

and simplifying yields the energy equation

$$\begin{aligned} h \zeta(x) + 4 \sigma \epsilon T(x)^3 \zeta(x) + 1.16 \frac{k_f}{d} (\text{Gr Pr})^{1/10} \zeta(x) \\ - \frac{k_{wi} d}{4} \frac{d^2 \zeta(x)}{dx^2} = \frac{I^2 r_e}{\pi d} (1 + \alpha_1 \zeta(x)) \end{aligned}$$

The boundary conditions are

$$\zeta(\pm \frac{\ell}{2}) = T_s - T_e \quad \text{and} \quad \frac{d\zeta(0)}{dx} = 0$$

Introducing the nondimensional variables

$$\xi = \frac{2x}{\ell} \quad \text{and} \quad a(\xi) = \alpha_1 \zeta(x)$$

simplifies the differential equation to

$$\begin{aligned} & \left[ \frac{hd}{k_f} + \frac{4 \sigma \epsilon T^3(\xi) d}{k_f} + 1.16(\text{Gr Pr})^{1/10} - \frac{\alpha_1 I^2 r_e}{\pi k_f} \right] a(\xi) \\ &= \frac{k_{wi}}{k_f} \frac{d^2}{\ell^2} \frac{d^2 a(\xi)}{d\xi^2} + \frac{\alpha_1 I^2 r_e}{\pi k_f} \end{aligned}$$

The first and last terms within the brackets (forced convection and ohmic heating) are of  $O(1)$ . The other two terms within the brackets (radiation and free convection) are of smaller order as shown and hence are negligible. Typical values for air flows at  $M = 2.5$  with overheat ratios of 1 and  $d = 0.0004''$  are  $T(\xi) \sim 1000^\circ \text{R}$ ,  $T_e \sim 500^\circ \text{R}$ ,  $k_f \sim 0.02 \text{ BTU/hr-ft-}^\circ\text{R}$ ,  $C_p = 0.25 \text{ BTU/lbm-}^\circ\text{R}$ ,  $\mu_f = 0.05 \text{ lbm/hr-ft}$ ,  $\rho_f = 0.006 \text{ lbm/ft}^3$ ,  $g = 32 \text{ ft/sec}^2$  and  $\beta = 0.002/^\circ\text{R}$ . The radiation term

$$\frac{4 \sigma \epsilon T^3(\xi) d}{k_f} \approx 10^{-2}$$

and the free convection term

$$1.16(\text{Gr Pr})^{1/10} \approx 0.2.$$

Therefore radiation and free convection are negligible provided that the overheat ratios are less than unity and wire diameters do not exceed  $0.0004''$ . If they are larger the free convection term must be considered. A theoretical discussion of free and forced convection of fine hot wires is given by Woods (22).

The governing differential equation reduces to

$$\frac{hd}{k_f} a(\xi) - \frac{\alpha_1 I^2 r_e}{\pi k_f} (1 + a(\xi)) = \frac{k_{wi}}{k_f} \frac{d^2}{\ell^2} \frac{d^2 a(\xi)}{d\xi^2}$$

with the boundary conditions

$$\frac{da(0)}{d\xi} = 0 \quad \text{and} \quad a(\pm 1) = \alpha_1 (T_s - T_e)$$

## 2.2 Nusselt Number Correction

The relation between the Nusselt number ( $hd/k_f$ ) for an infinitely long wire and a finite wire must be determined. Denoting the infinite wire by  $( )^*$

$$a^*(\xi) = a^* = \text{constant, i.e., no end losses.}$$

Therefore the equation becomes

$$h = \frac{\alpha I^2 r_e}{\pi d} \frac{1 + a^*}{a^*}$$

The average convective coefficient of a finite wire is

$$\bar{h} = \frac{\alpha I^2 r_e}{\pi d} \frac{1 + \bar{a}}{\bar{a}}$$

where  $\bar{a}$  = average non-dimensional temperature of the finite wire

and also

$$\bar{a} = \alpha_1 (T_w - T_e) = \frac{r_w - r_e}{r_e} \quad (\text{overheat ratio})$$

Therefore

$$\frac{Nu_\infty}{Nu_m} = \frac{h}{\bar{h}} = \frac{1 + a^*}{1 + \bar{a}} \frac{\bar{a}}{a^*}$$

which gives the Nusselt number of an infinite wire in terms of the measured (average) Nusselt number and  $\bar{a}/a^*$ . To determine  $\bar{a}/a^*$ , the differential equation is solved for  $a(\xi)$  and averaged over the wire length.

On substituting for  $h$  and  $\alpha_1 I_e^2 r_e / \pi d$ , in terms of  $\bar{h}$ ,  $\bar{a}$ , and  $a^*$ , and rearranging, the differential equation becomes

$$\frac{\bar{h} d}{k_f} \frac{\bar{a}}{1 + \bar{a}} \frac{\bar{a}}{a^*} (a(\xi) - a^*) = \frac{k_{wi}}{k_f} \frac{d^2}{\ell^2} \frac{d^2 a(\xi)}{d\xi^2}.$$

The measured Nusselt number  $Nu_m = \bar{h}d/k_f$  and letting  $S = d/\ell [(1 + \bar{a}/Nu_m)(k_{wi}/k_f)]^{1/2}$  obtains

$$\frac{a^*}{\bar{a}} S^2 \frac{d^2 a(\xi)}{d\xi^2} - a(\xi) + a^* = 0$$

which becomes with one more change of variables

$$z = \xi \frac{1}{S} (\bar{a}/a^*)^{1/2} \text{ and } A(z) = a(\xi) - a^*$$

$$\frac{d^2 A(z)}{dz^2} - A(z) = 0$$

with the boundary conditions

$$\frac{dA(0)}{dz} = 0 \quad \text{and} \quad A[\pm (\bar{a}/a^*)^{1/2} (1/S)] = \alpha_1 (T_s - T_e) - a^*$$

The solution is

$$A(z) = \frac{\alpha_1 (T_s - T_e) - a^*}{\cosh[(1/S)(\bar{a}/a^*)^{1/2}]} \cosh(z)$$

or



$$a(\xi) = a^* + \frac{\cosh[(1/S)(\bar{a}/a^*)^{1/2} \xi]}{\cosh[(1/S)(\bar{a}/a^*)^{1/2}]} [\alpha_1(T_s - T_e) - a^*]$$

Integrating with respect to  $\xi$  and averaging yields

$$\frac{\bar{a}}{a^*} = 1 - \frac{[1 - \frac{\alpha_1}{a^*} (T_s - T_e)]}{\frac{1}{S} (\bar{a}/a^*)^{1/2}} \tanh \left[ \frac{1}{S} (\bar{a}/a^*)^{1/2} \right]$$

Kovaszny (1) assumed that the wire support temperature  $T_s$  equals the cold wire equilibrium temperature  $T_e$  and also that  $T_e$  is unaffected by the end losses. With this assumption, the end loss correction factor for the Nusselt number is

$$\frac{\bar{a}}{a^*} = 1 - \frac{\tanh \left[ \frac{1}{S} (\bar{a}/a^*)^{1/2} \right]}{\frac{1}{S} (\bar{a}/a^*)^{1/2}}$$

The graphical solution of this relation is shown in Fig. 2a.

Dewey (10) included the effects of different wire support temperatures. These effects are also shown in Fig. 2a and are seen to be secondary so they can be neglected.

The equilibrium (adiabatic wall) temperature of finite wire as measured differs from that of an infinite wire due to the end heat conduction losses. This difference, ignored by Kovaszny (1), has been considered by Dewey (10). The equilibrium temperature ratio or recovery ratio end loss correction as a function of  $S$  is shown in Fig. 2b.

The recovery temperature correction requires knowledge of the flow properties of the flow which are as yet unknown. Therefore in this paper the two corrections are considered separately. The Nusselt number corrections are given by Kovaszny in Fig. 2a and the recovery temperature

corrections by Dewey in Fig. 2b. Applying the Nusselt number corrections with the measured recovery temperature, a first approximation of the flow properties, Mach and Reynolds numbers, is obtained from the heat transfer correlation curves described below. These then can be used to correct for the recovery temperature end loss effect. The corrected recovery temperature is then used in place of the measured recovery temperature and the entire procedure is repeated. The procedure for correcting the recovery temperature will be described after the discussion of the heat transfer correlation curves.

### 2.3 Nusselt -Mach-Reynolds Numbers Correlation

Theoretical analyses for predicting the heat transfer to infinitely long cylinders in the continuum flow and free molecule flow are available, but the transition between the two regimes must be obtained empirically. Since extensive empirical data have been published, no theoretical analyses will be presented here. The experimental data have been summarized and correlated by Dewey (10) in terms of Nusselt number of infinitely long cylinders as functions of the local free stream Mach and Reynolds numbers. These correlations are presented graphically and analytically.

The best correlations were found to exist if all the fluid properties were evaluated at the total free stream temperature, Laufer and McClellan (9). The correlation curves are shown in Fig. 3. The correlation equations are given in two parts, as shown,

$$Nu_{\infty}(Re_{\infty}, M_{\infty}) = Nu_{\infty}(Re_{\infty}, \infty) \phi(Re_{\infty}, M_{\infty})$$

where  $Nu(Re_{\infty}, \infty)$  is the Nusselt number of an infinite cylinder at  $M_{\infty} = \infty$ . (Subscript  $\infty$  on  $Nu$  refers to infinite cylinder and on

Re and M refers to the local free stream conditions. Re is based on wire diameter.)

$$Nu_{\infty}(Re_{\infty}, \infty) = Re_{\infty}^n (0.14 + N_1 + N_2)$$

$$n = 1 - \frac{Re_{\infty}^{0.6713}}{5.142 + 2 Re_{\infty}^{0.6713}}$$

$$N_1 = \frac{0.2302 Re_{\infty}^{0.7114}}{15.44 + Re_{\infty}^{0.7114}}$$

$$N_2 = \left( \frac{0.01569}{0.3077 + Re_{\infty}^{0.7378}} \right) \left( \frac{15}{15 + Re_{\infty}^3} \right)$$

and  $\phi(Re_{\infty}, M_{\infty})$  is the deviation of the Nu at  $M_{\infty}$  from the Nu at  $M_{\infty} = \infty$ .

$$\phi(Re_{\infty}, M_{\infty}) = 1 + \phi_1 \cdot \phi_2 \cdot \phi_3$$

$$\phi_1 = \frac{0.06309}{M_{\infty}} + 0.5701 \left[ \left( \frac{M_{\infty}^{1.222}}{1 + M_{\infty}^{1.222}} \right)^{1.569} - 1 \right]$$

$$\phi_2 = 1.834 - 1.634 \left( \frac{Re_{\infty}^{1.109}}{2.765 + Re_{\infty}^{1.109}} \right)$$

$$\phi_3 = 1 + \left[ 0.300 - \frac{0.065}{M_{\infty}^{1.67}} \left( \frac{Re_{\infty}}{4 + Re_{\infty}} \right) \right]$$

These correlation equations are valid only in the range

$$0.4 \leq M_{\infty} < \infty \quad \text{and} \quad 0.01 \leq Re_{\infty} \leq 1000.$$

For boundary layer investigations, correlations for  $M_{\infty} < 0.4$  are required, hence the following correlation equations were derived for

$$0 \leq M_{\infty} < 0.4 \quad .$$

The  $M_\infty = 0$  correlation, as obtained from the Oseen solution of Cole and Roshko (16) and the data of Hilpert (17) is

$$Nu_\infty(Re_\infty, 0) = 0.19 + 0.66 Re_\infty^{0.45} .$$

In the Mach range  $0 < M_\infty < 0.4$ , a weighted logarithmic average between the  $M_\infty = 0$  correlation above and the Dewey correlation at  $M_\infty = 0.4$  is used.

$$Nu_\infty = 10^{\{F \log[Nu_\infty(Re_\infty, 0)] + (1-F) \log[Nu_\infty(Re_\infty, 0.4)]\}}$$

where

$$F = [0.618 + (2.5 M_\infty)^{0.9}]^{-1} - 0.618 .$$

The extended correlation is valid only for the range

$$0 \leq M_\infty < 0.4 \quad \text{and} \quad 0.1 < Re_\infty < 300 .$$

Some of the original data, Dewey's correlation, and the new correlation are shown in Fig. 3.

Other correlations have been used, specifically the Yanta correlation (18) as used by Sfeir (3). Yanta modified the Dewey equations to extend  $M_\infty$  down to 0.1. However, the correlation equations as reported by Yanta and Sfeir produced no correlation at all for  $M_\infty \leq 0.3$ . This difficulty and the desire to extend the  $Nu_\infty - M_\infty - Re_\infty$  correlations to all Mach numbers required the addition to the Dewey equations for  $M_\infty < 0.4$ .

Measuring the local free stream static pressure, in addition to the hot wire measurements, the Reynolds number is obtained as a unique function of the Mach number

$$Re_\infty = \frac{P_\infty d}{\mu_t (R T_t)^{1/2}} M_\infty \left[ \left( 1 + \frac{\gamma-1}{2} M_\infty^2 \right)^\gamma \right]^{1/2} .$$

It can be seen then that the Mach number must be determined iteratively from the correlation equations.

#### 2.4 Recovery Temperature Correction

To obtain the first approximation of the Mach and Reynolds numbers, the measured recovery temperature,  $T_{em}$ , for the finite wire, was used in the procedure of the previous section. The correlation data are, however, valid only for infinite wires, hence the recovery ratio or temperature as measured must also be corrected for the end loss effects.

The procedure for correcting for the recovery ratio end loss as outlined by Dewey (10) is as follows:

The end loss recovery correction factor is

$$\begin{aligned} C_R &= \frac{T_e^*}{T_{em}} = \frac{\eta^*}{\eta_m} = [1 - \omega(\frac{T_s}{T_{em}})](1-\omega)^{-1} \\ &= [1 - \omega(\frac{\eta_s}{\eta_m})](1-\omega)^{-1} \end{aligned}$$

where

$T_{em}^*$  = cold wire recovery temperature of the infinite wire  
 $\omega$  = a parameter that depends primarily on  $S$  and is given in Fig. 2b,

and

$\eta = T/T_t$ , the temperature ratio or recovery ratio depending on  $T$ .

Since it is not feasible to experimentally determine the wire support temperature, a good approximation is

$$T_s = T_t \frac{1 + Pr^{1/2} \cdot \frac{\gamma-1}{2} M_\infty^2}{1 + \frac{\gamma-1}{2} M_\infty^2}$$

A normalized recovery ratio  $\bar{\eta}$  of a conducting cylinder is defined in terms of the free molecular and high Reynolds number continuum limits recovery ratio,  $\eta_f$  and  $\eta_c$ , respectively,

$$\bar{\eta} = (\eta - \eta_c) / (\eta_f - \eta_c) \quad .$$

The continuum limit and free molecular recovery ratios have been correlated by Morkovin (7) as

$$\eta_c = 1 - 0.050 \left( \frac{M_\infty^{3.4}}{1.175 + M_\infty^{3.5}} \right)$$

and

$$\eta_f - \eta_c = 0.2167 \left( \frac{M_\infty^{2.80}}{0.8521 + M_\infty^{2.80}} \right)$$

The normalized recovery ratio is empirically correlated in terms of the local free stream Knudsen number in Fig. 4. The equation of the universal curve shown in the figure is

$$\bar{\eta} = \left( \frac{Kn_\infty^{1.193}}{0.4930 + Kn_\infty^{1.193}} \right)$$

and

$$Kn_\infty = [(\pi\gamma/2)(M_\infty/Re_\infty)]^{1/2}$$

Sufficient information exists to determine  $\eta_m^*$  ( $= T_{em}^*/T_t$ ) given the  $M_\infty$  and  $Re_\infty$  from the first Nu correction and the Nu - M - Re correlation. The final  $Nu_\infty$ ,  $M_\infty$ ,  $Re_\infty$  and  $T_{em}^*$  are determined by repeating the Nu correction with the computed  $\eta_m^*$  (or  $T_{em}^*$ ) instead of the measured  $\eta_m$  (or  $T_{em}$ ). Usually less than five iterations are required to obtain 0.01% accuracy in  $\eta_m^*$ . Sfeir (3) claims the same accuracy with only three iterations.

## 2.5 Local Flow Direction

The Mach number obtained in the preceding section is the actual Mach number only if the hot wire is perpendicular to the flow. If the wire is not oriented perpendicularly to the flow then the Mach number obtained is only the component of the flow normal to the wire. By obtaining the Mach number components at two different wire orientations, the resultant Mach number and direction are obtained as follows:

The resultant local flow direction relative to some reference line (e.g., a wind tunnel centerline as in Fig. 5) is given by

$$\theta_{\infty} = \tan^{-1} \left( \frac{M_1 \sin \theta_2 - M_2 \sin \theta_1}{M_1 \cos \theta_2 - M_2 \cos \theta_1} \right)$$

and the resultant Mach number is

$$\begin{aligned} M_{\infty} &= M_1 / \cos \left( \frac{\pi}{2} - \theta_1 + \theta \right) \\ &= M_2 / \cos \left( \frac{\pi}{2} - \theta_2 + \theta \right) \end{aligned}$$

With the resultant local free stream Mach number, the remaining flow properties may be obtained

$$T_{\infty} = T_t / \left( 1 + \frac{\gamma-1}{2} M_{\infty}^2 \right)$$

$$\rho_{\infty} = p_{\infty} / RT_{\infty}$$

and

$$U_{\infty} = M_{\infty} (\gamma RT_{\infty})^{1/2}$$

This completes the basic hot wire anemometry theory. A more complete analysis required for the data reduction procedure would have to include an uncertainty analysis. No attempt was made to determine the uncertainty

except that due to the wire diameter. The wire diameters were very difficult to measure accurately since they are on the order of 0.0004 to 0.0001 inches. The manufacturer's nominal values are used and are  $\pm 3\%$  uncertain. The derivation of the uncertainty due to the wire diameter is given in the appendix.



### 3.0 COMPUTATIONAL PROCEDURE

In this section, the step by step computation procedure for reducing the hot wire data is outlined. The computer program (the listings are in the appendix) follows this procedure exactly. The procedure for obtaining the data will not be discussed.

1. Calibrate the wire to obtain the first temperature resistivity coefficient,  $\alpha_1$ ,

$$\alpha_1 = \frac{(r_2 - r_1)}{r_1(T_2 - T_1)} \quad .$$

Handbook values are used for the second temperature coefficient of resistivity,  $\alpha_2$ . The temperature of the wire is

$$T = T_1 + \frac{1}{\alpha_1} \frac{r - r_1}{r_1} - \frac{\alpha_2}{\alpha_1^3} \left( \frac{r - r_1}{r_1} \right)^2 \quad .$$

2. Determine effective wire length,  $\ell$ , from the manufacturer's unit length resistance at a reference temperature  $T_k$  and the actual measured resistance at  $T_k$

$$\ell = \frac{r}{k} \quad .$$

3. Also determine the wire diameter or use the manufacturer's nominal specification (see step 25 for the wire calibration).
4. Start the run and determine the cold wire measured equilibrium temperature, the heated wire temperature, and the total temperature,  $T_t$ .

$$T_{em} = T_1 + \frac{1}{\alpha_1} \frac{r_{em}-r_1}{r_1} - \frac{\alpha_2}{\alpha_1^3} \left( \frac{r_{em}-r_1}{r_1} \right)^2$$

$$T_{wm} = T_{em} + \frac{1}{\alpha_1} \frac{r_w-r_{em}}{r_{em}} - \frac{\alpha_2}{\alpha_1^3} \left( \frac{r_w-r_{em}}{r_{em}} \right)^2$$

5. Determine the viscosity and conductivity of air at the total temperature.

Sutherland's viscosity equation (Ref. 12)

$$\mu_t \times 10^7 = 7.323 T^{3/2} / (T + 198.72) \quad \left( \frac{\text{lbm}}{\text{ft-sec}} \right)$$

and conductivity

$$k_a = \frac{1.1408 \times 10^{-3} T^{3/2}}{T + (441.72/10^{21.6/T})} \quad \left( \frac{\text{BTU}}{\text{hr-ft-}^\circ\text{R}} \right)$$

6. Determine the thermal conductivity of the wire as a function of temperature (Ref. 14)

$$k_{wi} = k_{wi,1} + (T-T_1) \left( \frac{k_{wi,1} - k_{wi,2}}{T_1 - T_2} \right)$$

and compute  $k_{wi}$  at  $T = T_{wm}$ .

7. Measure the power dissipation in the wire

$$\dot{q}_m = I^2 r_{wm} = \frac{v^2}{r_{wm}}$$

where  $v$  = voltage drop across the hot wire and is given in terms of the bridge voltage,  $V$ , by

$$v = V \frac{r_{wm}}{(r_{\text{bridge}} + r_{\text{lead}} + r_{wm})}$$

8. Assume an equilibrium recovery factor or use the measured value

$$(\eta_m = T_{em}/T_t), \text{ and compute}$$

$$T_{em} = \eta_m T_t$$

9. Compute the air conductivity from step 5 with  $T = T_{em}$

$$k_{am} = f(T_{em})$$

10. Compute the measured Nusselt number

$$Nu_m = \frac{4.097 q_m}{\pi(T_{wm} - T_{em}) \ell k_{am}}$$

11. Calculate the end loss parameter,  $S$ ,

$$S = \frac{d}{\ell} \left( \frac{k_{wi}}{k_{am}} \frac{1}{Nu_m} \frac{r_{wm}}{r_{em}} \right)^{1/2}$$

and the overheat ratio

$$\frac{\bar{a}_w}{a_w} = \frac{r_{wm} - r_{em}}{r_{em}}$$

12. Read off the value for

$$\frac{\bar{a}_w}{a_w^*} = f(S)$$

from Fig. 2a or use the cubic polynomial

$$\frac{\bar{a}_w}{a_w^*} = 1 - 1.100 S + 0.0999 S^2 - 0.250 S^3$$

$$\text{hence } a^* = \bar{a}_w / (\bar{a}_w / a_w^*)$$

and the Nusselt number correction factor for the end loss

$$C_N = \frac{\bar{a}_w}{a_w} \left( \frac{1 + a_w^*}{1 + \bar{a}_w} \right)$$

Thus the corrected Nusselt number for an infinitely long wire is

$$Nu_\infty = C_N Nu_m$$

13. Assume  $M_\infty$  and with the measured  $p_\infty$ , compute  $Re_\infty$

$$Re_\infty = 68.067 \frac{p_\infty d}{\mu_t (RT_t)^{1/2}} M_\infty [\gamma (1 + \frac{\gamma-1}{2} M_\infty^2)]^{1/2}$$

14. With these values for  $M_\infty$  and  $Re_\infty$ , determine  $Nu_\infty$  from the correlation  $Nu - Ma - Re$  correlation curves as given in Fig. 2 and Section 2.3.

$$Nu_\infty = f(Re_\infty, M_\infty)$$

15. Compare this  $Nu_\infty$  with the  $Nu_\infty$  obtained in Step 12. Readjust  $M_\infty$  in Step 13 and recompute until

$$Nu_\infty(\text{step 12}) \equiv Nu_\infty(\text{step 14}).$$

16. Knowing the  $M_\infty$  and  $Re_\infty$ , compute the Knudsen number

$$Kn_\infty = \left( \frac{\pi \gamma}{2} \right) \frac{M_\infty}{Re_\infty},$$

the continuum recovery ratio

$$\eta_c = 1.0 - 0.05 \frac{M_\infty^{3.5}}{(1.175 + M_\infty^{3.5})},$$

the free-molecule flow recovery ratio

$$\eta_f = \eta_c + 0.2167 \left( \frac{M_\infty^{2.80}}{0.8521 + M_\infty^{2.80}} \right) ,$$

and the universal recovery factor

$$\bar{\eta} = \frac{Kn_\infty^{1.193}}{0.4930 + Kn_\infty^{1.193}} .$$

17. The recovery factor for the infinite wire is then

$$\eta = \bar{\eta} (\eta_f - \eta_c) + \eta_c$$

18. Compute the adiabatic wire support temperatures from  $M$

$$T_s = T_t (1 + (Pr)^{1/2} \frac{\gamma-1}{2} M_\infty^2) / (1 + \frac{\gamma-1}{2} M_\infty^2)$$

and the support temperature ratio

$$\eta_s = T_s / T_t$$

19. Read the ratio  $\omega/(1-\omega)$  as a function of  $S$  from Fig. 2b or compute from

$$\frac{\omega}{1-\omega} = 0.700 S + 1.125 S^2 + 4.250 S^3$$

and solve for  $\omega$

$$\omega = \frac{[\omega/(1-\omega)]}{1 + [\omega/(1-\omega)]}$$

20. The end loss recovery factor correction ratio is

$$C_R = \frac{[1 - \omega(\eta_s/\eta_m)]}{1 - \omega}$$

21. The "measured" wire recovery ratio is now computed

$$\eta_m' = \eta/C_R$$

22. Compare with the measured or assumed value of  $\eta_m$  of Step 8. If the two differ, then go back to Step 8 and repeat the entire procedure with the computed "measured" wire recovery ratio  $\eta_m'$ . Usually no more than five iterations are required to obtain  $\pm 0.01\%$  precision in  $M_\infty$ .
23. The normal  $M_\infty$  component has now been determined for one wire orientation. Repeating the entire procedure obtains another normal  $M_\infty$  component at a different wire orientation at the same point in the flow field. With the given wire orientation angles  $\theta_1$  and  $\theta_2$  and the computed normal  $M_\infty$  components, the resulting  $M_\infty$  direction and magnitude are

$$\theta = \tan^{-1} \left( \frac{M_1 \sin \theta_2 - M_2 \sin \theta_1}{M_1 \cos \theta_2 - M_2 \cos \theta_1} \right)$$

and

$$M_\infty = M_1 / \cos \left( \frac{\pi}{2} - \theta_1 + \theta \right)$$

24. The remaining flow properties are now computed from the resultant  $M_\infty$ ,

$$T_\infty = T_t / \left( 1 + \frac{\gamma-1}{2} M_\infty^2 \right) ,$$

$$\rho_\infty = \frac{144 p_\infty}{R T_\infty}$$

and

$$U_\infty = M_\infty (\gamma R T_\infty)^{1/2} .$$

25. If the wire diameter used in Step 3 is of doubtful accuracy, the wire diameter is calibrated by measuring a very accurately known  $M_\infty$  and adjusting the wire diameter until the computed  $M_\infty$  equals the known  $M_\infty$ .

This completes the procedure for reducing the hot wire data at one particular point in the flow field.

A final word of warning is necessary. The whole point of using the  $Nu - Ma - Re$  correlation curves is to eliminate the need for calibration curves. This avoids the danger of calibrating bad data into "good" data. Calibrating the wire diameter as in Step 25 reintroduces that danger, and any adjustments of wire diameter greater than the manufacturer's tolerances should be considered suspect. In other words, the source of any discrepancy lies elsewhere.

The following section outlines the input data required for the data reduction program listed in the appendix.

#### 4.0 DATA INPUT

The data input required for the data reduction program can be divided into three types: program control data, general tunnel and wire conditions, and the specific data output of the hot wire anemometer. All data are initialized either at zero or a default value, as shown below. The data read-in is controlled by Subroutine DATA (IEOF), shown in Appendix A.

Each group of data within each of the three types is identified by a preceding card containing an integer 1 through 20 in the first two columns, right justified. Thus there are 20 data groups. The only exception to this rule is the data group labeled by the integer 1. This card must initially contain another integer in Columns 3 to 5, right justified. The other 19 identifiers contain one integer only; Columns 3 to 5 must be blank, otherwise the program stops.

Decimal data following the identifier card must be formatted in six words per card of 10 columns per word and must be right justified if in exponential form. Again the exception to this rule is the data labeled by Integer 1. Here the format is 8 columns for the first word, and 10 columns for each of the remaining 5 words. The reason for this exception is required by the peculiarities of the remote teletype terminal used to record the data. There is another difference in the Integer 1 data. The card immediately following the label card must either be blank or contain alpha-numeric characters in Columns 1 - 72. The same characters are printed in the output as a title. Omitting this card will cause a program termination.

Integer data are only 2 words per card and must be right justified in Columns 1 - 2, and 3 to 5. There are only two groups requiring integer data, namely, the data labeled by integers 14 and 15. The data sets



labeled by integers 19 and 20 are blank and have been provided for future expansion of the program.

The three types of data required are described below, along with their default or initial values.

### Program Control Data

This type of data is used to control the accuracy required, maximum number of iterations allowed, number of data sets to be reduced, and special printout statements for diagnosis and reminders. In each of the group descriptions below the first line contains the data label integer and the second the variable names.

10

DNUIC, DETAM

where

DNUIC - fractional allowable error on the Nusselt number calculation in Subroutine MARENU. Default value = 0.002.

DETAM - fractional allowable error on the recovery factor calculation in Program HOTWIRE. Default value = 0.002.

11

EBLT - an estimation of the boundary layer thickness. Default value = 0.10. This variable has no effect on any computations, all it does is control the distance over which a certain number (KK, see below) of data points are read; printout is "estimated boundary layer thickness = EBLT."

26

12

DETAIL - a parameter to be set to non-zero if a detailed printout is required for diagnosis. Default value = 0.0 for no detailed printout.

14

NPROFL - the number of profiles or data sets to be reduced. used only if the flow direction is to be computed. No default value.

15

ITMARN, ITETAM

where

ITMARN - the maximum number of iterations allowed for sub-routine MARENU to determine the Nusselt number from the correlation curves. Default number = 10.

ITETAM - the maximum number of iterations allowed for program HOTWIR to compute the recovery ratio. Default value = 5.

Note that there are two controls on the required accuracy of Mach number and Nusselt number. If the maximum numbers of iterations are exceeded a printout "Solution not accurate" will occur along with the inaccurate solution.

#### Tunnel and Wire Data

These data specify the fluid, wire, and anemometer constants. The default values are for the fluid being air, and the wire a 0.0004 inch platinum - 10% rhodium wire. The bridge resistance of the hotwire anemometer is 50 ohms.

4

GAM, R, PR, CMU

where

GAM - specific heat ratio. Default value = 1.4.

R - gas constant. Default value = 53.61.

PR - Prandtl's number. Default value = 0.71.

CMU - fluid viscosity. No default. Computed internally using Sutherland's viscosity formula for air. If a different fluid is used, the Sutherland equation must be bypassed.

5

WD - wire diameter. Default value = 0.0004 inches.

7

RESIST - hotwire anemometer bridge resistance. Default = 50 ohms.

8

CALCON - parameter near 1 to calibrate wire diameter if necessary. See computational procedure step 25. Default value = 1.0.

9

WK, TK

where

WK - unit length resistivity of the wire at the given temperature. Default value = 57.25 ohms per inch.

TK - temperature at which the above resistivity is specified. Default = 527.7°R.

28

16

WIK1, TW1, WIK2, TW2

where

WIK1, WIK2 - thermal conductivity of the hot wire at two  
different temperatures. Default values = 43.35,  
42.47 BTU/hr-ft-°R.

TW1, TW2 - the corresponding temperatures. Default  
= 450, 720 °R.

17

ALFA, BETA

where

ALFA - temperature coefficient of resistance. Default  
=  $0.96\text{E-}3 \text{ }^{\circ}\text{R}^{-1}$ .

BETA - second temperature coefficient of resistance.  
Default =  $-0.462\text{E-}7 \text{ }^{\circ}\text{R}^{-2}$ .

18

TT - total temperature. Default 535 °R.

#### Specific Test Data

These data include output voltages of the anemometer, temperature-resistivity calibration, static pressure, total temperature, overheat and wire lead resistances, and the wire orientation angles.

1 K

(Alpha-numeric title or blank)

VOLTS(1), VOLTS(2), VOLTS(3), ... , ... , VOLTS(6)

VOLTS(7), ... , VOLTS(I), ... , VOLTS(K)

where

K - number of output voltages to be read for this particular data run,  $K \leq 200$ .

(title) - any alpha-numeric or blank statement to be printed as a title for this test run. If omitted, program terminates.

VOLTS(I) - the bridge voltage of the anemometer. Default = 0.0 and program termination.

2

PINF - static pressure. Must be obtained separately as it is not part of the hot wire anemometry instrumentation. Default value = 0.495 psi.

3

T1, WR1, T2, WR2

where

WR1, WR2 - the wire resistances as measured by the anemometer bridge at the corresponding temperatures. Default values = 2.6, 3.047 ohms.

T1, T2 - the corresponding temperatures. Default values = 492, 672 °R.

6

WRWM, RESISL

where

WRWM - overheat resistance. No default. Omission causes program termination.

RESISL - hot wire lead resistance as measured by the hot wire anemometer bridge. No default.

THETA1, THETA1, THETA2

where

THETA1 - parameter to be set to 1 or 2 if the program is to determine local flow direction. If THETA1 = 1., then data following is for THETA1, and if THETA1 = 2., data following for THETA2. Default = 0.0. If omitted, no angular calculation.

THETA1 - wire orientation or angle for the first Mach number component as shown in Figure 5. Default = 90°.

THETA2 - wire angle for the second Mach number. Default = 45°.

The execution starts after reading in VOLTS(K). Hence the data sets 2, 3, 6, 13 must precede set 1. If THETA1 is non-zero then two sets of data must be read in each with the same K. PINF need not be same but should not differ by more than 10%. The two data sets for the angles need not be together. A maximum number of profiles or data sets can be read in at one angle and then the profiles at the second angle may be read in. The only restriction is the number of profiles at each angle must be the same and the sequence of profiles must be in the same order for each angle.

## 5.0 CONCLUSIONS

The data reduction procedure is limited by the following restrictions:

- 1) fluid can be gases only, which approximate the ideal gas law,
- 2) overheat ratio  $\leq 0.5$ ,
- 3) Reynolds number based on wire diameter  $0.1 \leq Re_{\infty} \leq 300$ , and
- 4) wire diameter should be calibrated.

Other than these restrictions, the program is quite flexible. There are no restrictions on the Mach or Prandtl number.

The default values in the computer program for the fluid and wire properties are those for air and a 0.0004 inch diameter platinum-10% rhodium wire. These values may be changed if other gases and wires are used. The data input requirements are given in the previous section. The program listing is in Appendix A.

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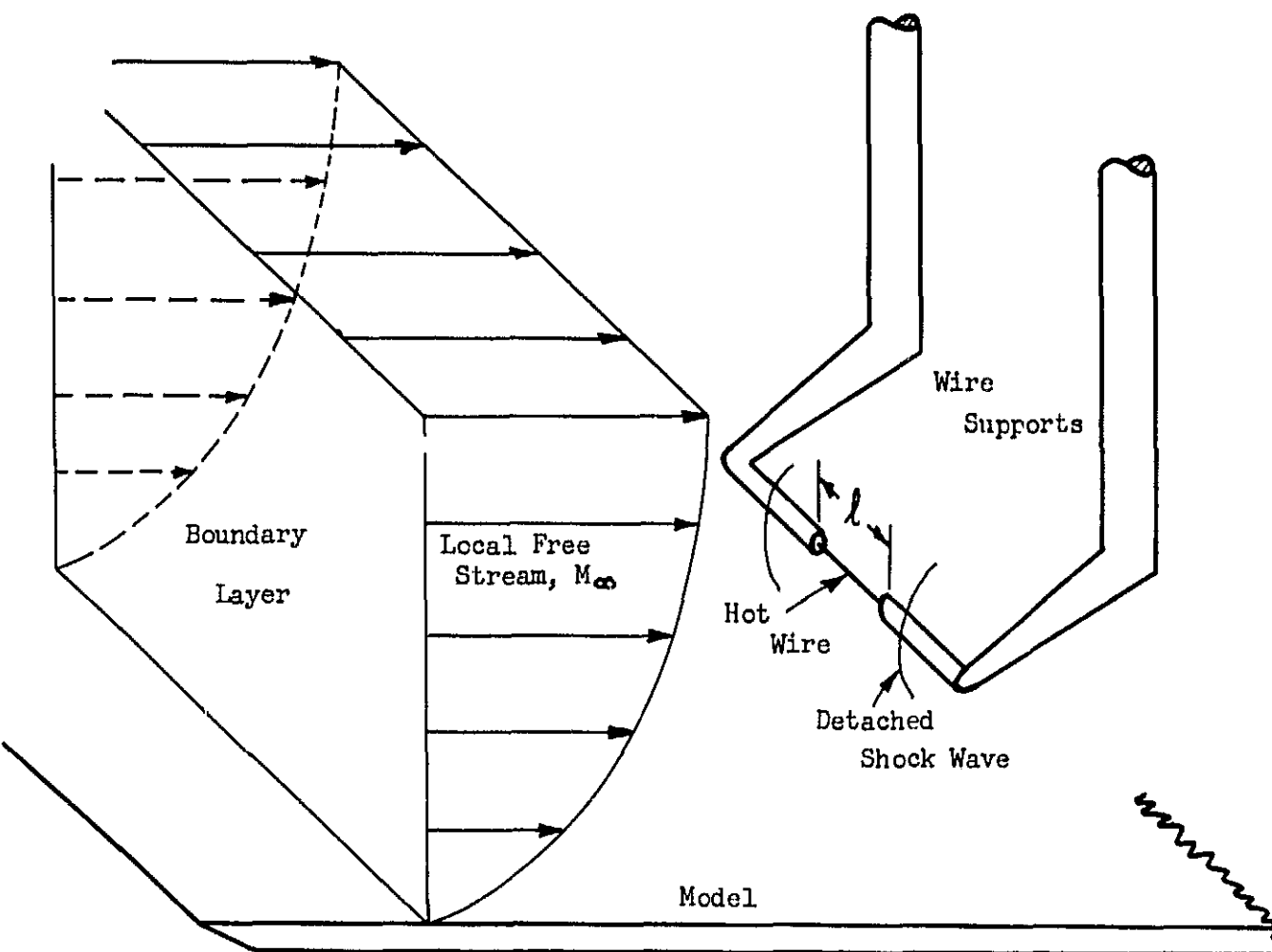


FIGURE 1a. Schematic of Hot Wire in the Boundary Layer of the Model.

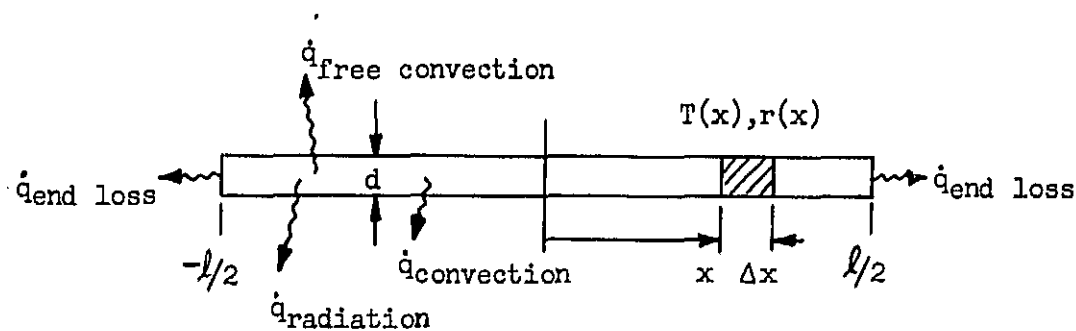


FIGURE 1b. Coordinate System and Heat Losses of the Hot Wire.

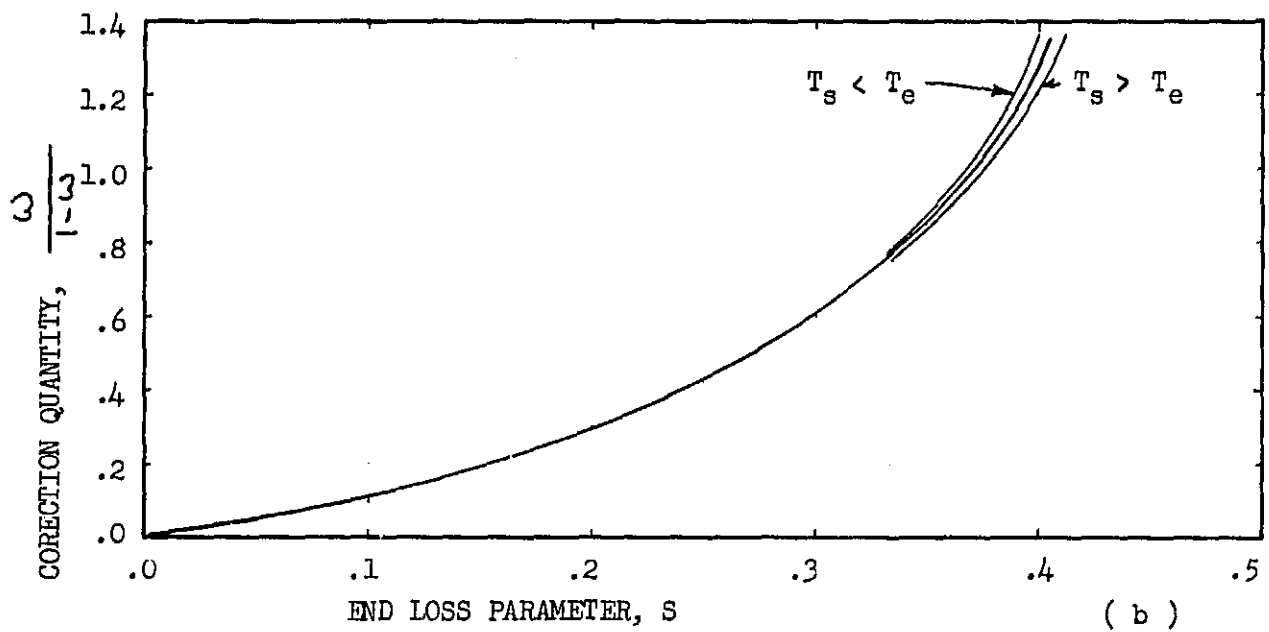
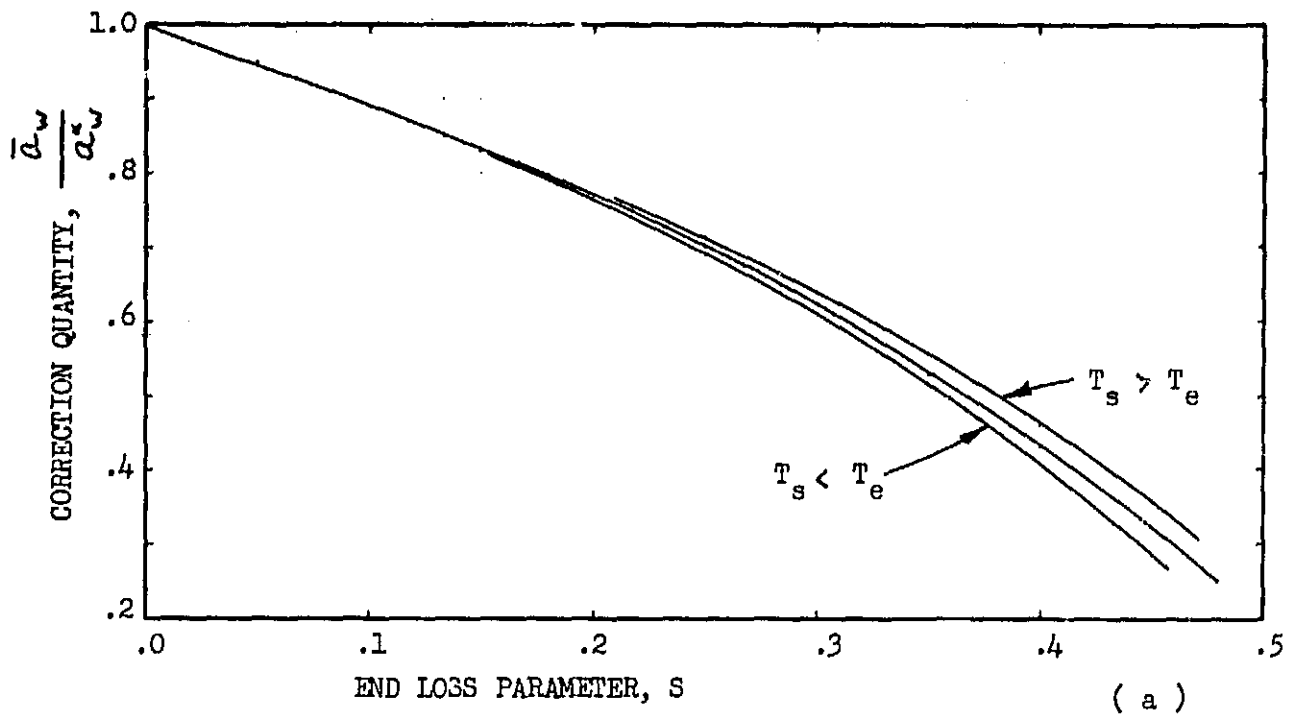


FIGURE 2. End loss correction for the Nusselt number ( a ) and recovery temperature ( b ), Reference 10.

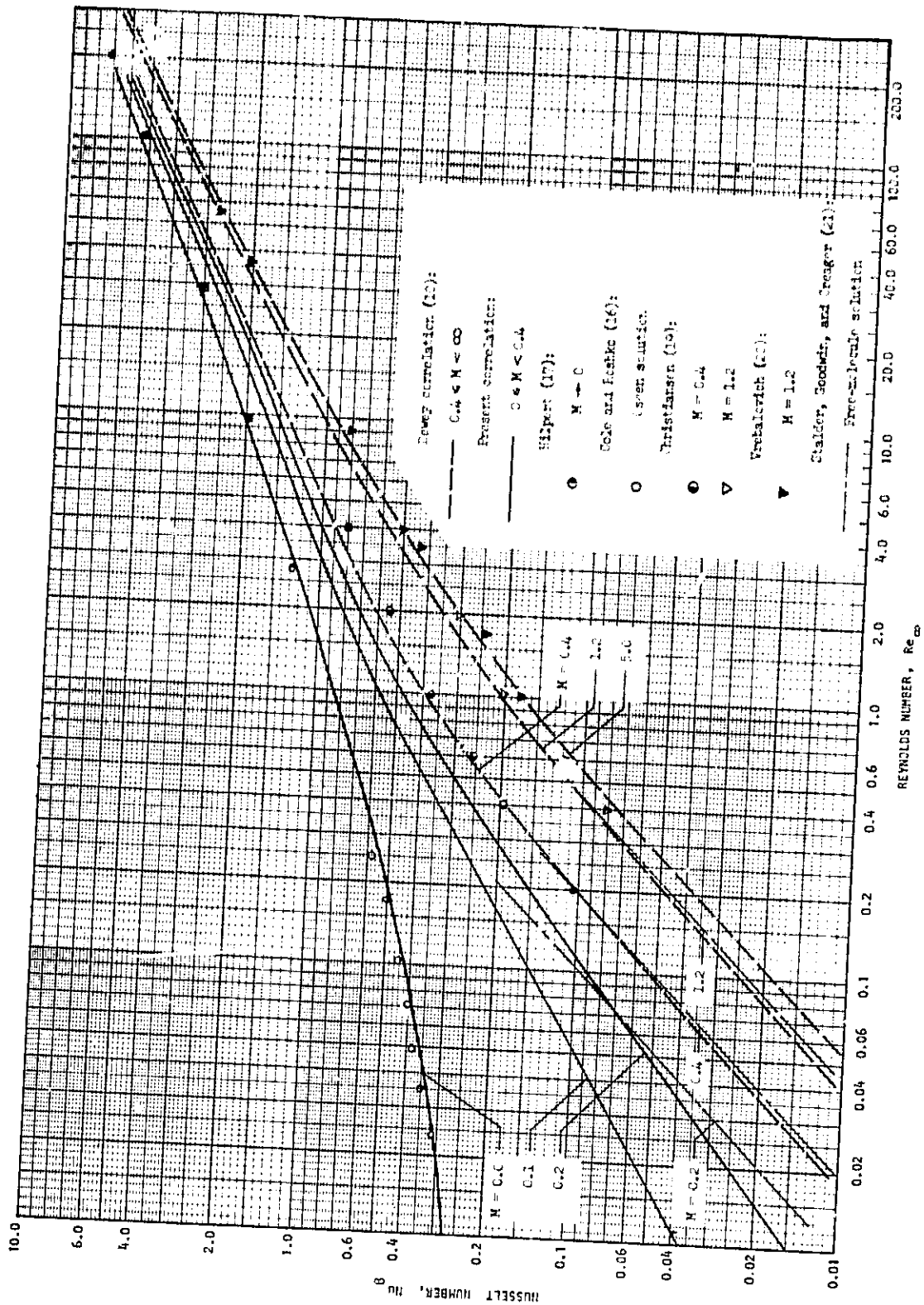


FIGURE 3. Empirical convective heat transfer correlation for cylinders

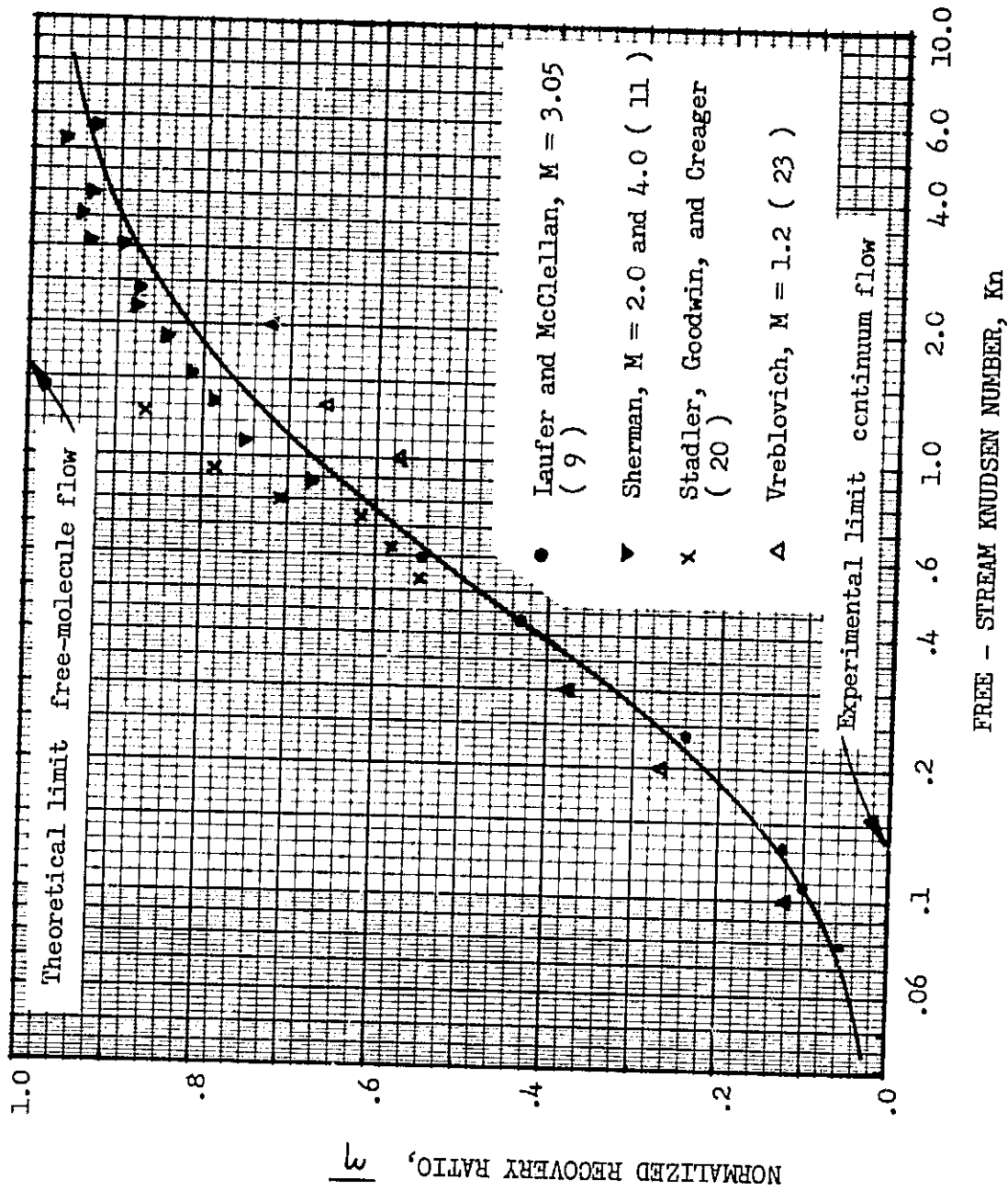


FIGURE 4. Free-Stream Knudsen Number Correlation Curve, ( reference 10 ).

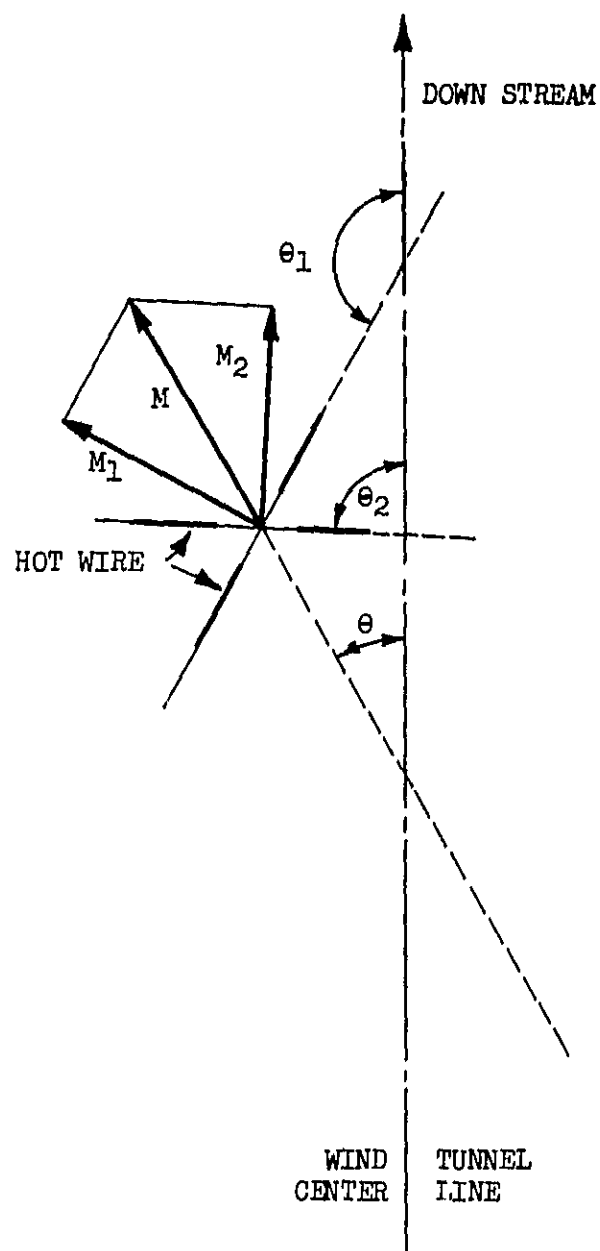


FIGURE 5. Coordinate System for the Local Flow Direction

APPENDIX A  
PROGRAM LISTINGS

PROGRAM HOTWIR(INPUT,OUTPUT,TAPE50 = INPUT)

PROGRAM HOT WIRE \*\*\* COMPUTES MACH NUMBERS AND VELOCITIES

LOGICAL IFOF

```
DIMENSION XMTH1(20,200),XMTH2(20,200),PINF1(20),PINF2(20)
COMMON/RESULT /XM,REI,CNUI,CKNI,CNUM,CNUIC
COMMON/ETA/ ETA,ETAC,ETAF,ETAS,ETABAR,ETAM,ETAM1,ETADEL
COMMON/CORCOE/ CN,CR
COMMON/ENDLOS/ S,AWBAR,AWSTAR,AWRA,OMEGA,OMGRA
COMMON/TUNNEL/PINF,TT,TINF
COMMON/WIRCAL/TW1,WIK1,TW2,WIK2,T1,WR1,T2,WR2,WK,TK,CALCON
COMMON/WIRE/WD,WL,ALFA,BETA,WIK,WREM,WRWM,WRM,TFM,TWM,DELT
COMMON/GAS/GAM,P,PR,CMU,RGAM
COMMON/VOLTS/VOLTS(200)
COMMON/RESIST/RESIST,RESISL
COMMON/KONST/K
COMMON/INDEX/IJI
COMMON/INDEXS/KJI,JJJ,LLL,ITMARN,ITETAM
COMMON/PRECIS/DNUIC,DETAM
COMMON/EBLT/FBLT
COMMON/DETAIL/DETAIL
COMMON/THETA1/THETA1,THETA1,THETA2
COMMON/NPROFL/NPROFL
```

```
DATA (VOLTS(I),I=1,200)/200*0.0/
DATA ALFA,BETA/.96E-3,-.462E-7/,TK,WK/527.7,57.25/
DATA WIK1,TW1/43.35,450./,WIK2,TW2/42.47,720./
DATA R,GAM,PR /53.61,1.4,0.710/
DATA T1,WR1,T2,WR2 /491.7,2.6,671.7,3.047/
DATA PINF,TINF,TT/.495 ,450.,535./
DATA WD,WL/0.0004,.0446/
DATA RESIST/50./
DATA RESISL/.24/
DATA DNUIC,DETAM/.002,.002/
DATA CALCON/1./
DATA FBLT/0.10/
DATA DETAIL/0.0/
DATA (PINF1(I),I=1,20)/20 *0.0/
DATA (PINF2(I),I=1,20)/20 *0.0/
DATA THETA1,THETA1,THETA2/ 0.0,90.,45./
DATA ITMARN,ITETAM/10.5/
```

NPTH1 = 0  
NPTH2 = 0

1 CONTINUE  
CALL DATA(IEOF)  
IF( IEOF ) GO TO 5000  
IF ( THETA1 .EQ. 1.) GO TO 15  
IF ( THETA1 .EQ. 2.) GO TO 16  
GO TO 4000

15 CONTINUE  
NPTH1 = NPTH1 + 1  
PINF1(NPTH1) = PINF  
GO TO 4000

16 CONTINUE  
NPTH2 = NPTH2 + 1  
PINF2(NPTH2) = PINF

4000 CONTINUE  
RGAM = (GAM - 1.)/2.  
SGAMPI = SQRT(GAM\*1.5708)  
GAMPC = 32.174\*R\*GAM

DETERMINE AIR THERMAL CONDUCTIVITY (BTU/HP-FT-R) AND  
VISCOSITY ( LBM/FT-SEC)

T = TT  
T15 = T\*\*1.5  
AKM = .0011408\*T15/(T + 441.72/(10.\*\*(21.6/T)))  
CMU = (7.323E-7)\*T15/(T + 198.72)

CALIBRATE

R1 = WR1  
R2 = WR2  
ALFA = (R2-R1)/(R1\*(T2-T1))  
WR = R1\*(1.+ALFA\*(TK-T1))  
WL = WR/WK  
PRINT 2000

DETERMINE MEASURED HEATED WIRE TEMPERATURE

TWM = T1+((WRWM-WR1)/(ALFA\*WR1))\*(1.-BETA\*(WRWM-WR1)/  
1 (WR1\*ALFA\*ALFA))  
WIK = WIK1+(TWM-TW1)\*(WIK1-WIK2)/(TW1-TW2)

XM = .05  
ASSUME VALUE FOR THE ETAM  
50 ETAM = 1.



```

DO 1000 I = 1, K
  JJJ      = 0
75 TEM     = ETAM*TT
  JJJ      = JJJ + 1
  DELT     = TWM-TEM
  WREM     = R1*(1.+(TEM-T1)*(ALFA + BETA*(TEM-T1)))
  WRM      = WRWM
  EWR      = WREM
  VOLT     = VOLTS(I)*WRWM/(WRWM+RESIST+RESISL)
  QM       = (VOLT*VOLT/WRWM)
  CNUM     = 13.042 * QM/(AKM*WL*DELT)
  S        = (WD/WL)*SQRT(WIK*WRM/(AKM*EWR*CNUM))*CALCON
  AWBAR    = (WRM-EWR)/EWR
  AWRA     = 1. + S*(-1.1+S*(.0999+S*(-2.25)))
250 AWSTAR = AWBAR/AWRA
  CN       = AWRA*(1.+AWSTAR)/(1.+AWBAR)
  CNU1     = CN*CNUM
  IJ1      = I
C
C SOLVE FOR MACH NUMBER VIA ITERATION
C
  KJ1      = 0
  CALL MARENU
  IF (DETAIL .EQ. 0.0) GO TO 13
  IF (IJ1 .NE. 1 .AND. IJ1 .NE. K) GO TO 13
C *****
  PRINT 10
10 FORMAT (* ALFA,WL,WIK,WRM,EWR,TT,TEM /DELT,ETAM,VOLT,QM,CNUM,S,
1 AWBAR/AWRA,AWSTAR,CN,CNU1,CNUIC,REI,XM *)
  PRINT 12, ALFA,WL,WIK,WRM,EWR,TT,TEM
  PRINT 12, DELT,ETAM,VOLT,QM,CNUM,S,AWBAR
  PRINT 12, AWRA,AWSTAR,CN,CNU1,CNUIC,REI,XM
12 FORMAT (7F14.4)
C *****
13 CONTINUE
  CKNI     = (XM/REI)*SGAMPI
  RM35     = XM**3.5
  ETAC     = 1.-.05*RM35/(1.175+RM35)
  RM28     = XM**2.8
  ETADEL   = .2167*RM28/(.8521+RM28)
  RK11     = CKNI**1.193
  ETABAR   = RK11/(.493+RK11)
  ETA      = ETABAR*ETADEL+ETAC
  RM2GAM   = RGAM*XM*XM
C
C COMPUTE SUPPORT RECOVERY RATIO = ETAS = TS/TT
C
  ETAS     = (1.+SQRT(PR)*RM2GAM)/(1.+RM2GAM)
  OMGRA    = S*(.7+S*(+1.125+S*11.25 ))
900 CONTINUE
  OMEGA    = OMGRA/(1.+OMGRA)

```

```

C
C SOLVE FOR FND LOSS RECOVERY FACTOR CORRECTION RATIO
C
C      CR      = (1.-OMEGA*(ETAS/ETAM))/(1.-OMEGA)
C
C COMPUTE FOR ETAM
C
C      ETAM1   = ETA/CR
C
C COMPARE COMPUTED AND ASSUMED ETA
C
C      IF(ABS((ETAM-ETAM1)/ETAM) .LE. DETAM) GO TO 950
920 CONTINUE
C      IF(JJJ.GT. ITETAM) GO TO 951
C      ETAM    = ETAM1 - (ETAM1-ETAM)/2.
C      GO TO 75
951 CONTINUE
C      PRINT 718
718 FORMAT (* SOLUTION NOT ACCURATE- MORE ETAM ITERATION NEEDED *)
950 CONTINUE
C      IF(LLJ.GT.ITMARN) PRINT 717
717 FORMAT (* SOLUTION NOT ACCURATE - MORE MARENU ITERATION NECY *)
C
C COMPUTE LOCAL FREE STREAM TEMPERATURE,DENSITY,MASS FLOW, AND VELOCITY
C
C      TINF    = TT/(1.+RM2GAM)
C      RHO     = 144.*PINF/(R*TINF)
C      V       = XM*SQRT(GAMRC*TINF)
C      IF ( DETAIL .EQ. 0.0) GO TO 14
C      IF ( IJI .NE. 1 .AND. IJI .NE. K ) GO TO 14
C *****
C      PRINT 7001
7001 FORMAT (* CN,CNUI,CKNI,ETAC,ETADEL,ETABAR,ETA/ETAS,OMGRA,OMEGA,
1 CR,ETAM1,TINF,PINF/WD,REI,R,RHO,T15,V,RM2GAM *)
C      PRINT 5001, CN,CNUI,CKNI,ETAC,ETADEL,ETABAR,ETA
C      PRINT 5001, ETAS,OMGRA,OMEGA,CR,ETAM,TINF,PINF
C      PRINT 5001, WD,REI,R,RHO,T15,V,RM2GAM
5001 FORMAT (7F14.4)
C      PRINT 6002
6002 FORMAT (* TWM,DELT,WIK,T,CNUIC,REI,XM *)
C      PRINT 5001, TWM,DELT,WIK,T,CNUIC,REI,XM
C *****
14 CONTINUE
C      AI      = 1
C      PRINT 980, AI,XM,V,RHO,TINF
980 FORMAT(1X,F3.0,E20.4,10X,F20.4,10X,E20.4,10X,E20.4)
2000 FORMAT (1H,10X,19H LOCAL MACH NUMBER,10X,20H LOCAL VELOCITY,F/S
1,10X,20H LOCAL DENSITY,L/CF,3X,32H LOCAL FREE STREAM TEMPERATU
PRE,R )
C      IF ( THETA1 .EQ. 1.) GO TO 1015
C      IF ( THETA1 .EQ. 2.) GO TO 1016
C      GO TO 1020

```

```

1015 CONTINUE
      XMTH1(NPTH1,1) = XM
      GO TO 1020
1016 CONTINUE
      XMTH2(NPTH2,1) = XM
1020 CONTINUE
      IF((NPTH1+NPTH2).EQ.(2*NPROFL) .AND. 1.EQ.K) GO TO 3000
1111 CONTINUE
1000 CONTINUE
      GO TO 1
3000 CONTINUE
      IF ( THETA1 .NE. 1. .AND. THETA1 .NE. 2.) GO TO 5000
      RTET1 = THETA1/57.296
      RTET2 = THETA2/57.296
      DO 8100 NP = 1,NPROFL
        IF ( PINF1(NP) .NE. PINF2(NP)) GO TO 1017
        PINF = PINF1(NP)
        GO TO 1018
1017 CONTINUE
        PINF = (PINF1(NP) + PINF2(NP))/2.
1018 CONTINUE
        ANP = NP
        PRINT 8000, ANP
8000 FORMAT (1H1,F3.0,* PROFILE * )
        PRINT 8001
8001 FORMAT(8X,32H LOCAL MACH NO.      ANGLE, DEG  ,24H LOCAL VELOCITY,
1 F/5      , 2X,21H LOCAL DENSITY, L/CF ,2X,35H LOCAL FREE STREAM TEMP
PERATURE,R      )
        DO 8200 J = 1,K
          AJ = J
          RTETA = ATAN((XMTH1(NP,J)*SIN(RTET2)- XMTH2(NP,J)*SIN(RTET1))/
1 (XMTH1(NP,J)*COS(RTET2) - XMTH2(NP,J)*COS(RTET1)))
          THETA = RTETA*57.296
          XM = XMTH1(NP,J)/COS(1.5708 - RTET1 + RTETA)
          RM2GAM = RGAM*XM*XM
          TINF = TT/(1. + RM2GAM)
          RHO = 144.*PINF/(R*TINF)
          V = XM*SQRT(GAMRC*TINF)
          PRINT 8002, AJ,XM,THETA,V,RHO,TINF
8002 FORMAT(1X,F3.0,E15.4,12X,F5.1,3X,E15.4,7X,E20.4,10X,E20.4)
8200 CONTINUE
8100 CONTINUE
      NPTH1 = 0
      NPTH2 = 0
      THETA1 = 0.0
      THETA2 = 0.0
      GO TO 1
5000 CONTINUE
      STOP
      END

```

## SUBROUTINE MAPENU

```

COMMON/RESULT /XM,REI,CNUI,CKNI,CNUM,CNUIC
COMMON/ETA/ ETA,ETAC,ETAF,ETAS,ETABAR,ETAM,ETAM1,ETADEL
COMMON/CORCOE/ CN,CR
COMMON/ENDLOS/ S,AWBAR,AWSTAR,AWRA,OMEGA,OMGRA
COMMON/TUNNEL/PINF,TT,TINF
COMMON/WIRCAL/TW1,WIK1,TW2,WIK2,T1,WR1,T2,WR2,WK,TK,CALCON
COMMON/WIRE/WD,WL,ALFA,BETA,WIK,WREM,WRWM,WRM,TEM,TWM,DELT
COMMON/GAS/GAM,R,PR,CNU,RGAM
COMMON/VOLTS/VOLTS(200)
COMMON/KONST/K
COMMON/INDEX/IJ1
COMMON/RESIST/RESIST,RESISL
COMMON/INDEX/KJ1,JJJ,LLL,ITMARN,ITFTAM
COMMON/PRECIS/DNUIC,DETTAM
COMMON/EBLT/EBLT

```

```

LLL      = 0
Y0       = ALOG10(CNUI)
DLX      = 0.1
X1       = ALOG10(XM)
KKK      = 1
GO TO 100
1  CONTINUE
  IF(ABS((CNUIC-CNUI)/CNUIC) .LE. DNUIC) GO TO 600
  Y1      = ALOG10(CNUIC)
1000 CONTINUE
  X2      = X1 + DLX
  KKK     = 2
  XM      = 10.**X2
  GO TO 100
2  CONTINUE
  LLL     = LLL + 1
  IF( LLL .GT. ITMARN ) GO TO 600
C  COMPARE CNUIC AND CNUI AND RETURN IF GOOD
  IF(ABS((CNUIC-CNUI)/CNUIC) .LE. DNUIC) GO TO 600
  Y2      = ALOG10(CNUIC)
  DLX     = (Y0 - Y2)*(X2 - X1)/(Y2 - Y1)
  X1      = X2
  Y1      = Y2
  GO TO 1000

```

100 CONTINUE

REI = 68.06655\*(XM\*PINF\*WD/CMU)\*CALCON\*

1 SORT(GAM\*(1. + RGAM\*XM\*XM)/(R\*TT))

C IF XM LESS THAN .4 THEN AN AVERAGE IS TAKEN OF THE VALUES OF CNUIC  
C AT XM = 0.0 AND XM = .4

IF(XM=.4) 400,400,450

400 CONTINUE

R6 = REI\*\*0.6713

SN = 1.-(R6)/(5.142+2.\*R6)

R7 = REI\*\*0.7114

CN1 = 0.2302\*R7/(15.44+R7)

CN2 = (.01569/(.3077+REI\*\*.7378))\*(15./(15.+REI\*\*3.))

PHI1 = 1.00283

R11 = REI\*\*1.109

PHI2 = 1.834 - 1.634\*(R11/(2.765+R11))

PHI3 = 1. - .000242\*(REI/(4. + REI))

PHI = 1. + PHI1\*PHI2\*PHI3

CNUIM4 = (.14+CN1+CN2)\*(REI\*\*SN)\*PHI

CNUIMO = .19 + .66\*REI\*\*.45

FF = -.618034 + 1./((XM/.4)\*\*.9 + .618034)

CNUIC = 10.\*\*((FF\*ALOG10(CNUIMO) + (1. - FF)\*ALOG10(CNUIM4))

GO TO 500

450 CONTINUE

R6 = REI\*\*0.6713

SN = 1.-(R6)/(5.142+2.\*R6)

R7 = REI\*\*0.7114

CN1 = 0.2302\*R7/(15.44+R7)

CN2 = (.01569/(.3077+REI\*\*.7378))\*(15./(15.+REI\*\*3.))

RM12 = XM\*\*1.222

PHI1 = .6039/XM + .5701\*((RM12/(1. + RM12))\*\*1.569 - 1.)

R11 = REI\*\*1.109

PHI2 = 1.834 - 1.634\*(R11/(2.765+R11))

PHI3 = 1.+(.3-.065/(XM\*\*1.67))\*(REI/(4.+REI))

PHI = 1. + PHI1\*PHI2\*PHI3

CNUIC = (.14+CN1+CN2)\*(REI\*\*SN)\*PHI

500 CONTINUE

GO TO (1,2),KKK

600 RETURN

END

SUBROUTINE DATA(IEOF)

LOGICAL IEOF

COMMON/RESULT /XM,REI,CNUI,CKNI,CNUM,CNUIC  
 COMMON/ETA/ ETA,ETAC,ETAF,ETAS,ETABAR,ETAM,ETAMI,ETADEL  
 COMMON/CORCOE/ CN,CR  
 COMMON/ENDLOS/ S,AWBAR,AWSTAR,AWRA,OMFGA,OMGRA  
 COMMON/TUNNEL/PINF,TT,TINF  
 COMMON/WIPCAL/TW1,WIK1,TW2,WIK2,T1,WR1,T2,WR2,WK,TK,CALCON  
 COMMON/WIRE/WD,WL,ALFA,BETA,WIK,WPEM,WRWM,WPM,TEM,TWM,DELT  
 COMMON/GAS/GAM,R,PP,CMU,RGAM  
 COMMON/VOLTS/VOLTS(200)  
 COMMON/KONST/K  
 COMMON/RESIST/RESIST,RESISL  
 COMMON/INDEXS/KJI,JJJ,LLL,ITMARN,ITETAM  
 COMMON/PREFIS/DNUIC,DETAM  
 COMMON/EBLT/EBLT  
 COMMON/DETAIL/DETAIL  
 COMMON/THETA1/THETA1,THETA1,THETA2  
 COMMON/NPROFL/NPROFL

1000 CONTINUE

READ 100,II,KK

100 FORMAT ( 12,I5)

IF ( EOF,50 ) 2000,101

101 CONTINUE

IEOF = .FALSE.

IF (KK.GT. 0) K = KK

C  
 C KK SHOULD BE READ IN ONLY IF II = 1 AND IF NUMBER OF  
 C VOLTAGE READINGS PER RUN CHANGES

GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20),II

C  
 C NOTE ALL UNITS ARE AS FOLLOWS  
 C RESISTANCES OHMS  
 C TEMPERATURES DEGREES RANKINE  
 C LENGTHS INCHES  
 C VISCOSITY LBM/FT-SEC  
 C GAS CONSTANT FT-LBF/LBM-R  
 C CONDUCTIVITY BTU/HR-FT-R

C  
 C A CARD MUST PRECEDE EACH DATA BLOCK TO LABEL THE FOLLOWING DATA  
 C THE CARD PRECEDING THE VOLTAGE DATA CONTAINS TWO INPUTS, II AND K  
 C THE LABEL CARD FOR ALL OTHER DATA BLOCKS CONTAINS ONLY ONE INPUT, II

C  
 C II = 1 SHOWS THE VOLTAGES OF ANFMOMETER READINGS  
 C K = NUMBER OF VOLTAGE READINGS TAKEN FOR THAT RUN  
 C  
 C II = 2 LOCAL FREE STREAM STATIC PRESSURE, PINF  
 C II = 3 WIRE CALIBRATIONS T1,WR1,T2,WR2  
 C II = 4 GAS DATA GAM,R,PR,CMU  
 C II = 5 WIRE DIAMETER, WD  
 C II = 6 HEATED WIRE RESISTANCE,WRWM , AND LEAD RESISTANCE,RESISL  
 C II = 7 BRIDGE RESISTANCE R3,RESIST  
 C II = 8 CALIBRATION CONSTANT, CALCON  
 C II = 9 WIRE UNIT LENGTH RESISTIVITY(OHMS/IN.),WK, AT TEMP., TK  
 C II = 10 PRECISION ON NUSSELT'S NO. AND EQ. TEMP. RATIO,DNUIC,DFTAM  
 C II = 11 ESTIMATED B.L. THICKNESS, EBLT, (IN.)  
 C II = 12 PARAMETER TO BE SET TO NON-ZERO IF A DETAILED PRINTOUT IS  
 C REQUIRED, DFTAIL. THE P.O. WILL BE FOR THE FIRST AND LAST  
 C RUNS ONLY.  
 C II = 13 PARAMETER TO BE SET TO 1. OR 2. IF ANGLE DEPENDENCE IS REQD  
 C THETA1 = 1. FOR 1ST ANGLE  
 C THETA1 = 2. FOR 2ND ANGLE  
 C AND THE VALUES OF THOSE TWO ANGLES, THETA1, THETA2  
 C II = 14 NUMBER OF PROFILES TO BE READ IN AT EACH ANGLE,NPROFL  
 C II = 15 NUMBER OF ITERATIONS TO COMPUTE NU,ITMARN, AND COMPUTED  
 C RECOVERY RATIO,ITETAM  
 C II = 16 THERMAL CONDUCTIVITIES OF WIRE AT TWO TEMPERATURES,  
 C WIK1,TW1,WIK2,TW2  
 C II = 17 FIRST AND SECOND TEMPERATURE COEFFICIENT OF RESISTIVITY,  
 C ALFA AND BETA  
 C II = 18 TOTAL TEMPERATURE,TT  
 C  
 C ALL DATA MUST BE RIGHT JUSTIFIED  
 C  
 C

```

1  READ 200
200 FORMAT(70H
1
  IF ( EOF,50 ) 2000,201
201 CONTINUE
  IEOF = .FALSE.
  PRINT 300
300 FORMAT (1H1)
  PRINT 200
  PRINT 350
350 FORMAT (1H )
  READ 401,(VOLTS(I),I=1,K)
401 FORMAT (F8.4,5F10.4)
400 FORMAT (6F10.4)
  RETURN
2  READ 400, PINF
  GO TO 1000
3  READ 400, T1,WR1,T2,WR2
  GO TO 1000
4  READ 400, GAM,R,PR,CMU
  GO TO 1000
  
```

```
5  READ 400, WD
   GO TO 1000
6  READ 400, WPWM,RESISL
   GO TO 1000
7  READ 400, RESIST
   GO TO 1000
8  READ 400, CALCON
   GO TO 1000
9  READ 400, WK,TK
   GO TO 1000
10 READ 400, DNUIC,DETAM
   GO TO 1000
11 READ 400, FBLT
   PRINT 982, FBLT
982 FORMAT(* ESTIMATED BOUNDARY LAYER THICKNESS = *,F6.3, *INCHES*)
   GO TO 1000
12 READ 400, DETAIL
   GO TO 1000
13 READ 400, THETA1,THETA1,THETA2
   GO TO 1000
14 READ 100, NPROFL
   GO TO 1000
15 READ 100, ITMARN,ITETAM
   GO TO 1000
16 READ 400, WIK1,TW1,WIK2,TW2
   GO TO 1000
17 READ 400, ALFA,BETA
   GO TO 1000
18 READ 400, TT
   GO TO 1000
19 CONTINUE
   GO TO 1000
20 CONTINUE
   GO TO 1000
2000 CONTINUE
    IEOF = .TRUE.
    RETURN
    END
```



APPENDIX B  
NON-LINEAR RESISTIVITY EFFECTS

The effects of neglecting the second temperature coefficient of resistivity are considered in this appendix.

The resistance of the wire at temperature  $T_w$  is

$$r_w = r_0 [1 + \alpha_1(T_w - T_0) + \alpha_2(T_w - T_0)^2]$$

where

$r_0$  = resistance at temperature  $T_0$

$\alpha_1$  = first temperature resistivity coefficient (linear)

$\alpha_2$  = second temperature resistivity coefficient.

Rewriting in terms of overheat ratio

$$\frac{r_w - r_0}{r_0} = \alpha_1(T_w - T_0) + \alpha_2(T_w - T_0)^2$$

and solving for  $(T_w - T_0)$

$$T_w - T_0 = \frac{1}{\alpha_1} \frac{r_w - r_0}{r_0} \quad \text{if } \alpha_2 = 0$$

$$T_w - T_0 = \frac{1}{2} \frac{\alpha_1}{\alpha_2} \left[ \left( 1 + \frac{4\alpha_2}{\alpha_1^2} \frac{r_w - r_0}{r_0} \right)^{1/2} - 1 \right] \quad \text{if } \alpha_2 \neq 0$$

expanding the bracketed term with

$$\left| \frac{4\alpha_2}{\alpha_1^2} \left( \frac{r_w - r_0}{r_0} \right) \right| < 1$$

yields

$$T_w - T_0 = \frac{1}{\alpha_1} \frac{r_w - r_0}{r_0} - \frac{1}{4} \frac{\alpha_2}{\alpha_1^3} \left( \frac{r_w - r_0}{r_0} \right)^2 + \frac{1}{32} \frac{\alpha_2^2}{\alpha_1^5} \left( \frac{r_w - r_0}{r_0} \right)^3 + \dots$$

therefore the error in the temperature due to neglecting the non-linear effect  $\alpha_2$  is

$$\frac{\alpha_1 r_0 (T_w - T_0)}{r_w - r_0} - 1 = \frac{1}{4} \frac{\alpha_2}{\alpha_1^2} \left( \frac{r_w - r_0}{r_0} \right) - \frac{1}{32} \frac{\alpha_2^2}{\alpha_1^4} \left( \frac{r_w - r_0}{r_0} \right)^2 + \dots$$

The temperature error for typical values of  $|\alpha_2| \sim 10^{-7}$  and  $\alpha_1 \sim 10^{-3}$  (Ref. 2) is 1% for an overheat ratio of 0.4 and 2.5% for an overheat of 1.0.

The non-linear temperature coefficient of resistivity may be safely neglected if the overheat ratio does not exceed 0.4.

## APPENDIX C

### UNCERTAINTY ANALYSIS OF THE WIRE DIAMETER

The effects of the wire diameter uncertainties are considered here. The uncertainties of all the required input data should be considered in determining the uncertainty of the output of the computational procedure of Section 3.0, i.e., Mach number. However, the diameter of the wire is the most uncertain of all the data inputs, and furthermore, the diameter is not readily measurable, either directly or indirectly. Usually the manufacturer's nominal diameter is used (accuracy  $\pm 3\%$ ). The remaining data can be determined rather accurately, hence their uncertainties will not be considered.

The uncertainty of a function,  $\Delta f/f$  (e.g., Mach number), is defined by Kline and McClintock (13) as

$$\begin{aligned}\frac{\Delta f}{f} &= \left[ \sum_{i=1}^N \left( \frac{x_i}{f} \frac{\partial f}{\partial x_i} \right)^2 \left( \frac{\Delta x_i}{x_i} \right)^2 \right]^{1/2} \\ &= \left[ \sum_{i=1}^N \left( x_i \frac{\partial \ln f}{\partial x_i} \right)^2 \left( \frac{\Delta x_i}{x_i} \right)^2 \right]^{1/2}\end{aligned}$$

where  $f = f(x_i)$ , and the  $x_i$ 's and  $(\Delta x_i/x_i)$ 's are the independent data inputs (and their respective uncertainties) required to determine  $f$ .

The uncertainty of mass flow due to uncertainty of wire diameter is determined as follows

$$Nu = A (Re)^{1/2} + B$$

where for

incompressible flow  
(King's Eq., Ref. 1)

$$A = 0.690 \quad B = 0.318$$

and

supersonic flow

$$A = 0.580$$

$$B = -0.795$$

$$Nu = \frac{hd}{k} \quad ; \quad Re = \frac{\rho U d}{\mu}$$

From the defining uncertainty equation

$$\rho U = \left( \frac{hd}{kA} - \frac{B}{A} \right)^2 \frac{\mu}{d}$$

$$\therefore \ln \rho U = \ln \mu - \ln d + 2 \ln \left( \frac{hd}{kA} - \frac{B}{A} \right)$$

$$\mu \frac{\partial \ln \rho U}{\partial \mu} = 1$$

$$d \frac{\partial \ln \rho U}{\partial d} = \frac{hd + Bk}{hd - Bk} = \frac{Nu + B}{Nu - B}$$

$$\therefore \frac{\Delta \rho U}{\rho U} = \left[ \left( \frac{\Delta \mu}{\mu} \right)^2 + \left( \frac{Nu + B}{Nu - B} \right)^2 \left( \frac{\Delta d}{d} \right)^2 \right]^{1/2}$$

For typical values in the subsonic range

$$\left( \frac{\Delta \rho U}{\rho U} \right)_{M \ll 1} = \pm [(0.01)^2 + \left( \frac{2 + 0.318}{2 - 0.318} \right)^2 (0.10)^2]^{1/2}$$

$$\approx \pm 14\% \quad \text{if} \quad \frac{\Delta d}{d} = \pm 10\%$$

$$\text{and} \quad \frac{\Delta \mu}{\mu} = \pm 1\%$$

$$\text{and} \quad Nu = 2$$

For typical values in the supersonic range

$$\left( \frac{\Delta \rho U}{\rho U} \right)_{M>1} = \pm [(0.01)^2 + \left( \frac{8 - 0.795}{8 + 0.795} \right)^2 (0.10)^2]$$

$$\approx \pm 8\% \text{ if } \frac{\Delta d}{d} = \pm 10\%$$

$$\text{and } \frac{\Delta \mu}{\mu} = \pm 1\%$$

$$\text{and } Nu = 8$$

Therefore the uncertainty in the mass flow is of the same order as the uncertainty in the wire diameter. Since the mass flow is directly proportional to the Mach number, the uncertainties in Mach number have the same values. Hence an effort must be made to correct for uncertain wire diameters.